Eggert, Gustafsson, and Rommer Reply to the "Comment on 'Phase diagram of an impurity in the spin-1/2 chain: two channel Kondo effect versus Curie law' "

The Comment [1] introduces the particular point at $J_1 \to \infty$ as a new fixed point in the phase diagram of the J_1-J_2-impurity model which was analyzed in our recent Letter [2]. The point at $J_1 \to \infty$ is certainly worth a separate discussion which we will give here, but our analysis comes to rather different conclusions than Zvyagin [1].

At $J_1 \to \infty$ the three spins $\vec{S}_0$, $\vec{S}_1$, and $\vec{S}_N$ are strongly coupled and form a complex of total spin $s = 1/2$, which is characterized by a *triplet* between the spins $\vec{S}_1$ and $\vec{S}_N$, which in turn is antiferromagnetically correlated with $\vec{S}_0$. This three-spin complex is therefore *not* decoupled from the rest of the chain, but the effective coupling is given by $2J/3$ instead. The point at $J_1 \to \infty$ therefore does *not* represent a fixed point at all since it does not correspond to a simple boundary condition in the spin chain. This is in contrast to the fixed point $\mathrm{O_{N-2}} \otimes \frac{1}{2}$ at $J_2 \to \infty$ where the three-spin complex is indeed decoupled from the chain since the spins $\vec{S}_1$ and $\vec{S}_N$ form a *singlet*. The effective coupling there is given by $-J^2 J_1/4J_2^2$ from a *third* order perturbation expansion, which is always irrelevant because this coupling is suppressed by one additional power of the cutoff due to the virtual excitations.

For simplicity we will label the point at $J_1 \to \infty$ by $\mathrm{P_{N-1}}$ because three spins are effectively replaced by one spin-1/2 complex which remains coupled to the chain. This point $\mathrm{P_{N-1}}$ is again characterized by a logarithmically diverging impurity susceptibility and a ferromagnetic correlation $\langle \vec{S}_1 \cdot \vec{S}_N \rangle > 0$, just like the fixed point $\mathrm{P_{N+1}}$. We therefore do not expect any phase transitions or any discontinuities in the order parameter between the two points.

As far as the field theory description near the fixed point $\mathrm{P_{N+1}}$ is concerned, we must emphasize again that the only leading irrelevant operator is given by $\partial_x \mathrm{tr} g$ [3]. The operators mentioned in the Comment [1] are not present at $\mathrm{P_{N+1}}$, since the impurity spin $\vec{S}_0$ has been absorbed in the chain and $\vec{S}_0$ can therefore not be used as an independent degree of freedom to construct operators.

In conclusion we find that the point $\mathrm{P_{N-1}}$ is not a fixed point and it appears to be in the same phase as $\mathrm{P_{N+1}}$. The phase diagram as shown in Fig. 1 is therefore complete and correct.

The comparison to the integrable impurity model [4] made in the Comment [1] is certainly interesting. However, obviously the integrable impurity model lives in a different parameter space, so that a direct comparison of the impurity susceptibilities may not be very meaningful. An impurity in a free Fermion model (xx-model) also gives only limited insight since the scaling dimensions are fundamentally different in interacting systems. However,

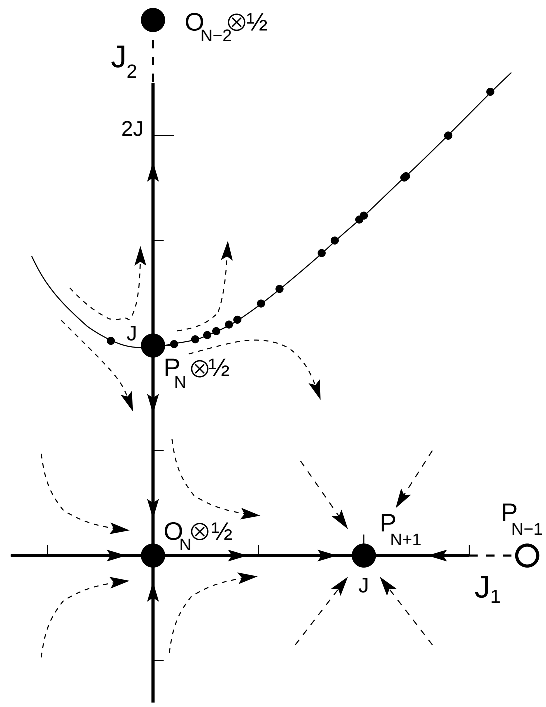

FIG. 1. The full phase diagram of the J_1-J_2-impurity model. Black dots are fixed points. The point $\mathrm{P_{N-1}}$ is indicated by a circle and does not represent a fixed point of the model.

it would be interesting to examine the integrable model [4] in more detail with field theory methods in an expanded parameter space, which, however, is not as trivial as indicated in the Comment [1]. This would answer the question if it may belong into one of the phases that were discussed in our Letter [2] or if it may correspond to a non-generic unstable multi-critical point as has been found for a related integrable impurity model [5].

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